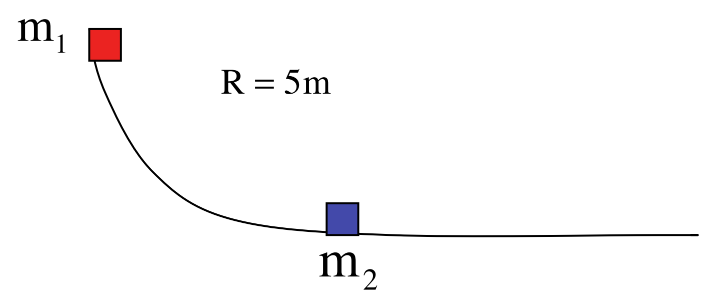
Problem 6.54

4 kg mass m_1 is released from top of curved incline (see sketch). It makes a head-on elastic collision with the 10 kg mass m_2. After the collision, how far up the incline will m_1 travel before coming to rest?



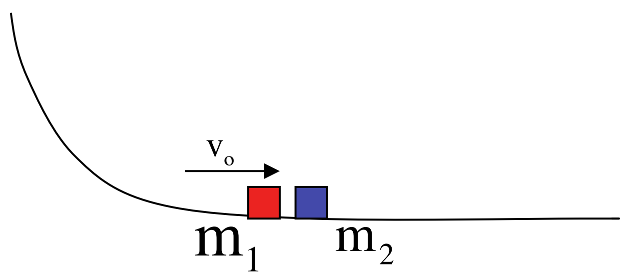
What do we know is true about:

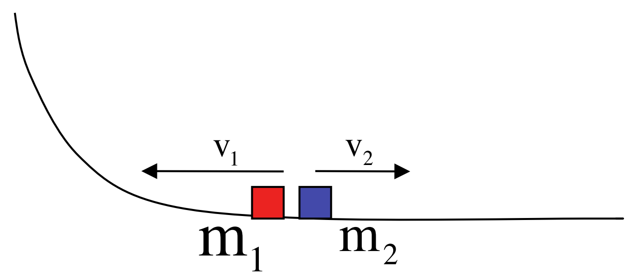
a.) energy considerations?

b.) momentum considerations?

To solve the problem:

Using cons. of momentum thru the collision where there are no external forces acting (hence no external impulses):





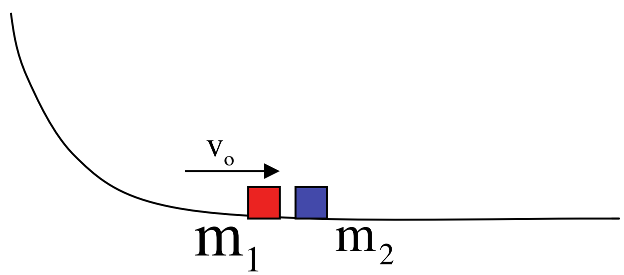
6.54)

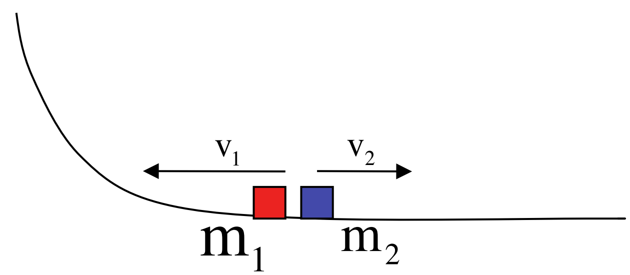
To solve the problem:

Using cons. of momentum thru the collision where there are no external forces acting (hence no external impulses):

$$\sum p_{1,x} + \sum F_{ext,x}\Delta t = \sum p_{2,x}$$

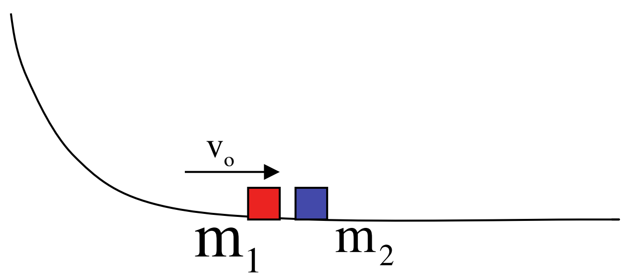
$$\Rightarrow m_1 v_o + 0 = -m_1 v_1 + m_2 v_2$$

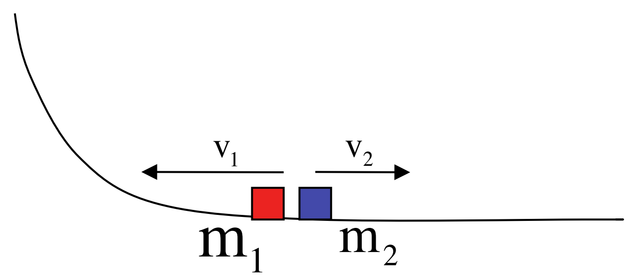




This leaves us with one equation and the three unknowns: v_o, v_1 and v_2

Momentum is not conserved before or after the collision. Under normal conditions, energy would be conserved up to the collision and directly after the collision, but not through the collision. This collision is different, though. We were told it is an ELASTIC collision. That means energy IS conserved THRU THE COLLISION and, for the “just before” and “just after” time interval, we can write:





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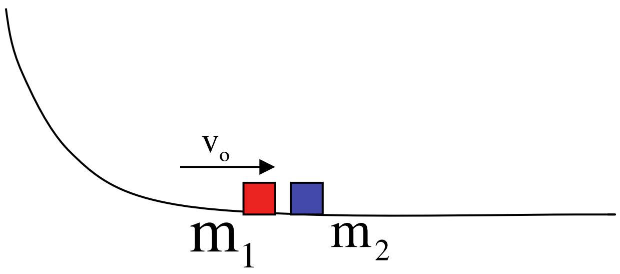
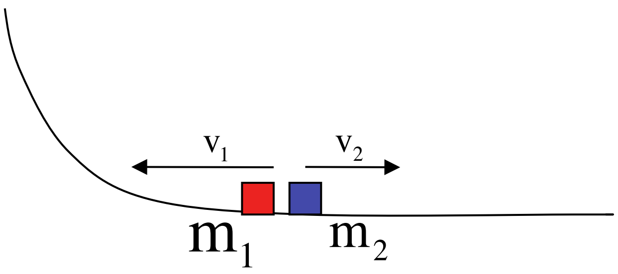
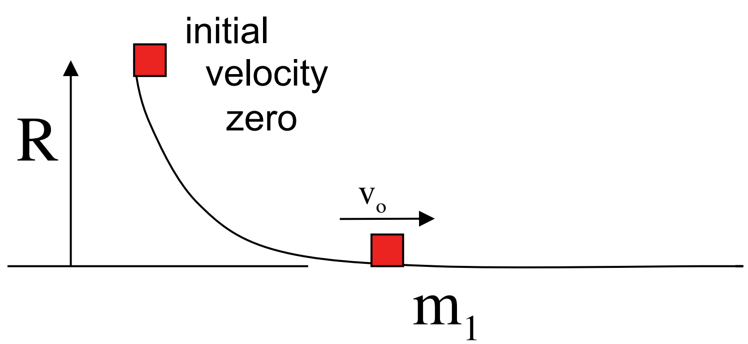
$$\sum KE_1 + \sum U_1 + \sum W_{extraneous} = \sum KE_2 + \sum U_2$$

$$\Rightarrow \frac{1}{2}m_1 v_o^{\,2} + 0 + O = \left(\frac{1}{2}m_1 v_1^{\,2} + \frac{1}{2}m_2 v_2^{\,2}\right) + 0$$





To get a third equation, we could consider mass m_1 as it moved from its initial position down through to the bottom of the arc (i.e., to just before the collision). During that motion, its energy is conserved so we can write:



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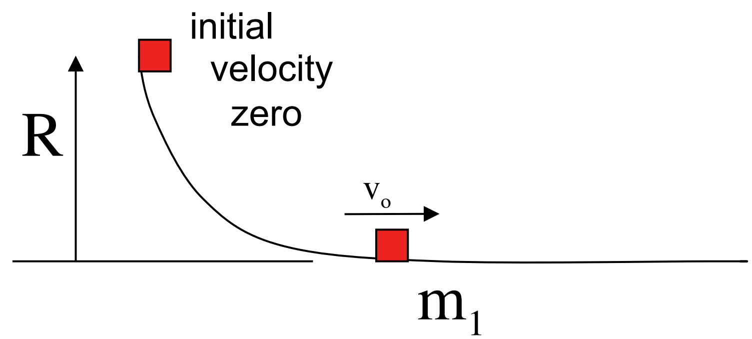
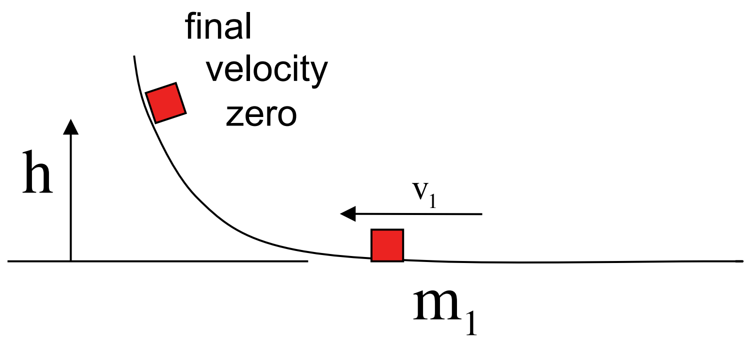
$$\sum KE_1 + \sum U_1 + \sum W_{extraneous} = \sum KE_2 + \sum U_2$$

$$\Rightarrow 0 \quad + m_1gR + \quad 0 \quad = \frac{1}{2}m_1v_o^{\ 2} + \ 0$$

$$\Rightarrow \ v_o = (2gR)^{1/2}$$



To get a last equation, we could consider mass m_1 as it moved from just after the collision to it's final resting place somewhere up the incline. During that motion, its energy is conserved so we can write:



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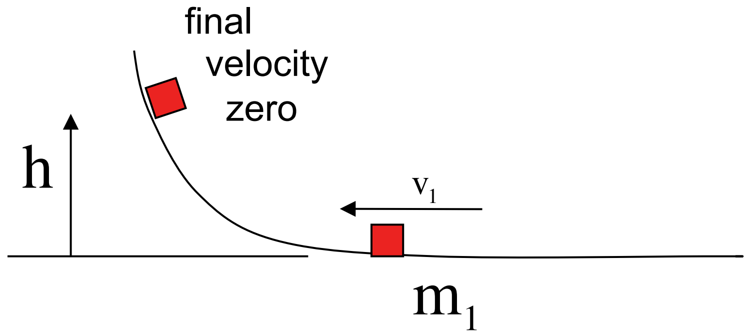
$$\sum KE_1 + \sum U_1 + \sum W_{extraneous} = \sum KE_2 + \sum U_2$$

$$\Rightarrow \frac{1}{2}m_1 v_1^{\,2} + 0 + O = 0 + m_1 gh$$

$$\Rightarrow h = \left(\frac{v_1^{\,2}}{2g}\right)$$



To summarize:

$$m_1 v_o = -m_1 v_1 + m_2 v_2 \qquad \text{A}$$

$$\frac{1}{2} m_1 v_o^{\,2} = \left(\frac{1}{2} m_1 v_1^{\,2} + \frac{1}{2} m_2 v_2^{\,2} \right) \qquad \text{B}$$

$$h = \left(\frac{v_1^{\,2}}{2g} \right) \qquad \text{C}$$

$$v_o = (2gR)^{1/2} \qquad \text{D}$$

We are pretty much done with equations C and D. You might think that the wisest way to deal with equations A and B is to put in numbers, then use a substitution approach. In fact, there is a better way. Follow along:

Begin by rearranging both equations by grouping all of the m_1 terms and all of the m_2 terms. Thus:

$$m_1 v_o = -m_1 v_1 + m_2 v_2$$

$$\Rightarrow m_1 (v_o + v_1) = m_2 v_2 \qquad \text{E}$$

and, after dividing out the 1/2's:

$$\frac{1}{2} m_1 {v_o}^2 = \left(\frac{1}{2} m_1 {v_1}^2 + \frac{1}{2} m_2 {v_2}^2 \right)$$

$$\Rightarrow m_1 ({v_o}^2 - {v_1}^2) = m_2 {v_2}^2$$

$$\Rightarrow m_1 (v_o - v_1)(v_o + v_1) = m_2 {v_2}^2 \qquad \text{F}$$

Dividing equation F by equation E yields:

Dividing equation F by equation E yields:

$$\frac{m_1(v_o\text{-}v_1)(v_o+v_1)}{m_1(v_o+v_1)} = \frac{m_2 {v_2}^2}{m_2 v_2}$$

Dividing out the m's and the one common velocity group yields:

$$(v_o\text{-}v_1) = v_2$$

Substituting this back into the momentum equation yields:

$$m_1 v_o = -m_1 v_1 + m_2(v_o - v_1)$$

$$\Rightarrow v_1 = \frac{\left[(-m_1 + m_2)\right] v_o}{(m_1 + m_2)}$$

Substituting in for v_o yields:

$$v_1 = \frac{\left[(-m_1 + m_2)\right](2gR)^{1/2}}{(m_1 + m_2)}$$

Putting v_1 into the "h" relationship, we get (next page):

$$h = \left(\frac{v_1{}^2}{2g} \right)$$

$$= \frac{\left(\frac{\left[(-m_1 + m_2)\right](2gR)^{1/2}}{(m_1 + m_2)} \right)^2}{2g}$$

$$= \frac{\left(\frac{(-m_1 + m_2)^2 (2gR)}{(m_1 + m_2)^2} \right)}{2g}$$

$$= \frac{(-m_1 + m_2)^2}{(m_1 + m_2)^2} R$$

$$= \frac{(-4 + 10)^2}{(4 + 10)^2} (5)$$

$$= .918 \text{ meters}$$

So what did we do? We used conservation of momentum through the collision to get equation A, conservation of energy through the collision to get equation B, conservation of energy from the collision to the end point to get equation C and conservation of energy from the start point to just before the collision to get equation D. Then we solved the mess for "h."

NICE, HUH?

$$m_1 v_o = -m_1 v_1 + m_2 v_2 \quad \text{(A)}$$

$$\frac{1}{2} m_1 {v_o}^2 = \left(\frac{1}{2} m_1 {v_1}^2 + \frac{1}{2} m_2 {v_2}^2 \right) \quad \text{(B)}$$

$$h = \left(\frac{{v_1}^2}{2g} \right) \quad \text{(C)}$$

$$v_o = (2gR)^{1/2} \quad \text{(D)}$$